The emergence of spacetime: Hints, questions, and progress

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The trouble with Gravity

D=4: $F = GM^2/r^2$,
dimensionless coupling $GE^2/\hbar c^5$
grows with E. Quantum
fluctuations are a problem,
growing at short distances.

Growth must(?) stop somewhere (non-trivial fixed point).

A priori options:

1) Fixed point contains gravity.
Still have dynamical smooth spacetime.
"Asymptotic Safety Scenarios;" analog of a CFT.
No tractable examples, and lots of questions.
2) Gravity "disappears" at fixed point, but smooth spacetime remains. Gravity "emerges" as an effective theory at low E.
Local versions forbidden (e.g. by Weinberg-Witten).
3) Spacetime disappears at short distances, replaced by something else. Spacetime "emerges" at low E.

No Gravity from local lattices

Recall the Newtonian Gauss Law

$$\oint \boldsymbol{g} \cdot \boldsymbol{n} = -4\pi\, GM$$

$$M \rightarrow E$$

E can be written as a boundary term at infinity.

Requires non-locality,
or implies bulk of lattice irrelevant.
Spacetime is emergent!

Summary of part I

Ideas for taming gravity's bad behavior seem to be

1. Impossible: Gravity emerges locally from some local non-gravitational theory, perhaps on a lattice.
2. Intractable: Asymptotic safety, or (strong) non-locality.
3. Spacetime is emergent too!

Part II, Black holes: Gravity & Thermodynamics

Simple question: How to add energy to a black hole?

- Shove it.
- Spin it.
- i.e., do obvious *Work* on it.

Note: We can undo these, and extract the energy back out of the black hole. Reversible!

But also: Just drop something in. BH grows! Not reversible: Hawking area thm says $dA/dt \geq 0$.

All changes in E are of one type or the other:

$$dE = \frac{k}{8\pi G}\, dA + Work\ Terms$$

$1/redshift \sim k\ (distance\ from\ horizon)$

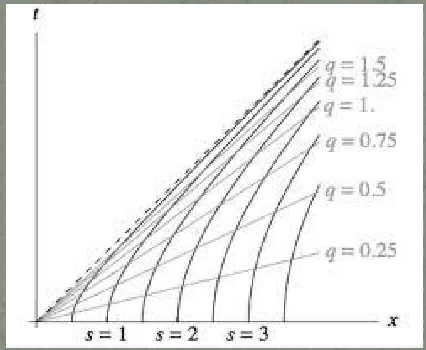
Hawking:
$T = \hbar\kappa/2\pi$

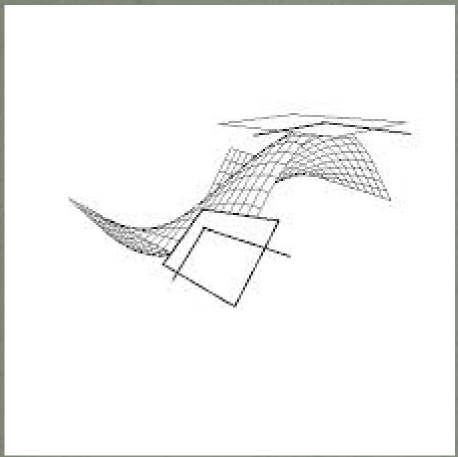
So $S = A/4G\hbar$ and we have the usual first & 2nd laws.

Gravity from thermo?

Jacobson 1995

(Rindler) Horizons exist in flat space as well.





And any spacetime is locally flat.

So there are local Rindler horizons
at every point of every spacetime.

In GR, they also satisfy $dE = TdS$ w/ $S = A/4G$.
(can choose local horizons s.t.work terms to vanish).

Jacobson's Result: This argument can be reversed!
Starting with $dE = TdS$ & $S = A/4G$ for each local horizon,
one can recover the full gravitational equations of motion.

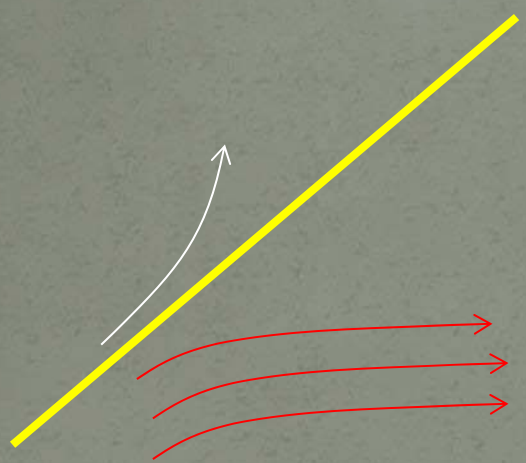
Suggests emergence

BH info problem

Ancient + Mathur, Giddings, AMPS, etc.

Flat Space QFT:

Vacuum is highly entangled. $< \varphi(x)\varphi(y) > \neq 0$
even for $\sigma^2 = -(x-y)^2 > 0$ (spacelike). Large for σ small.



A BH horizon is an *unstable* null surface.
Entanglement flows from short distances to long distances – eventually to far from the black hole.

Imposing a cut-off length scale, the entanglement visible above this scale grows with time.
But entanglement requires phase space!
So problem when it exceeds S_{BH}!
Occurs when area drops by factor of 2. (Page)

Resolving the problem requires modifying the state (naively in the UV), or modifying dynamics.

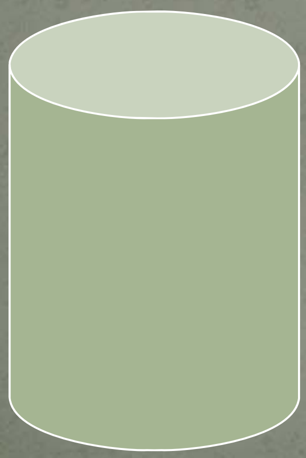
Is a new structure being revealed from which spacetime emerges in more familiar settings?

Part III: AdS/CFT

String theory gives us examples where gravity and bulk spacetime emerge from a theory without these concepts.

Gauge/gravity duality. Poster child is AdS/CFT (Maldacena). Also Matrix Theory (BFSS) and other (related) examples.

For AdS/CFT, spacetime in the gravitating theory has negative cosmo constant. Asymptotic to (locally) AdS_n x X for some compact X. AdS has curvature scale L and maximal symmetry.



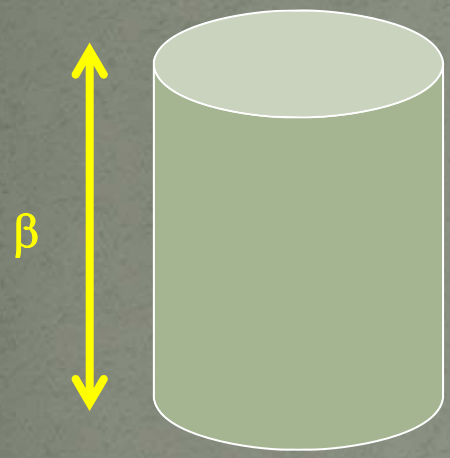
The AdS part has a (conformal) boundary B, with well-defined (conformal) metric.
This boundary is infinitely far from the interior.

The bulk is dual to a CFT on the conformal spacetime B.
Conformal group is dual to (asymptotic)AdS isometries.

Idea (Witten): Bulk path integral w/ given BCs on B is dual to a CFT path integral over B w/ sources.
$L^{D-2}/G_{bulk} = (L/L_{planck}) \sim N^b$
Classical bulk obtained in large N limit.

A simple example (Witten)

Consider the CFT partition function $Z = \mathrm{Tr}\, e^{-\beta H}$ on S^{d-1}.



Euclidean (imaginary time) path integral w/ periodic BCs.

Defines BCs for corresponding Euclidean bulk PI.

$$\int \exp(-G^{-1} I)$$

For G → 0 we evaluate this using saddle points; i.e., classical solutions to the bulk Euclidean EOMs.

Empty AdS has (Euclidean) time-translation symmetry. So one solutions is given by identifying the classical solution with period β.

$I = -\ln Z = \beta F$

$F = E - TS = E_0$ (constant)

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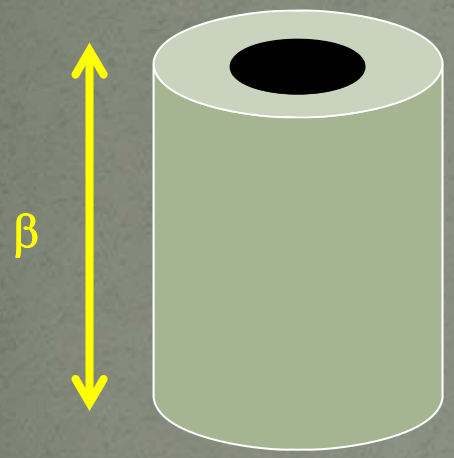
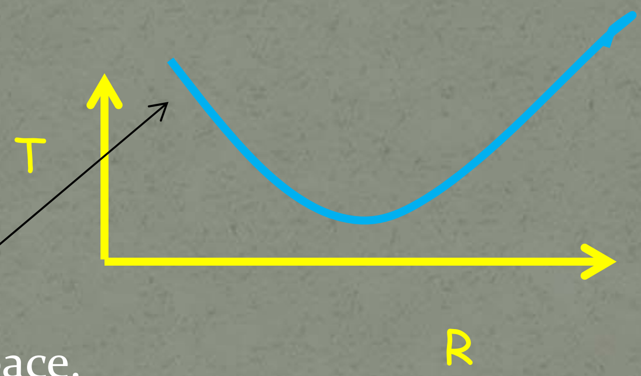


Black Hole solutions are also periodic in Euclidean time (thermal!). So (Euclidean) BH w/ $T = 1/\beta$ is also a solution.



R << L
BH as in flat space.
Only relevant scale is R,
so $R \sim \beta \sim 1/T$.

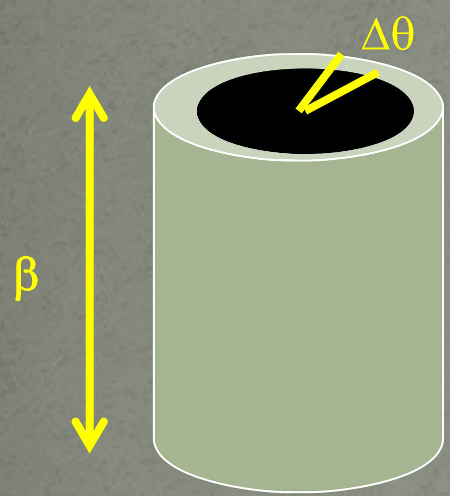
Horizon approaches bndy. Inverse redshift at bndy fixed as BC. But horizon forces a zero. So gradient becomes large. Proportional to κ and thus T.

$I = -\ln Z = \beta F = \beta E_{BH} - S_{BH}$

Clearly smaller for "large" BH at given T than for "small" one.

A simple example

Consider the CFT partition function $Z = \mathrm{Tr}\, e^{-\beta H}$ on S^{d-1}.



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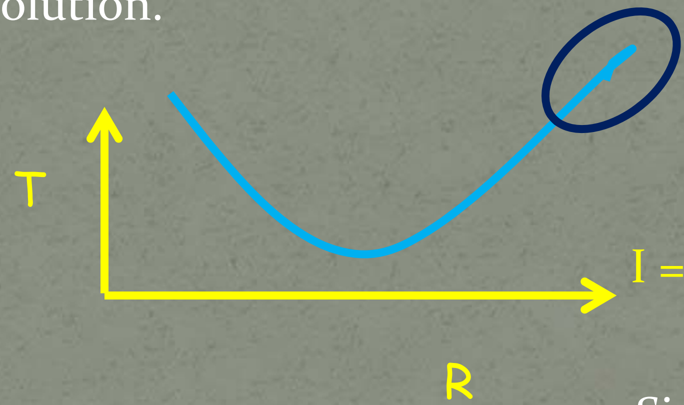
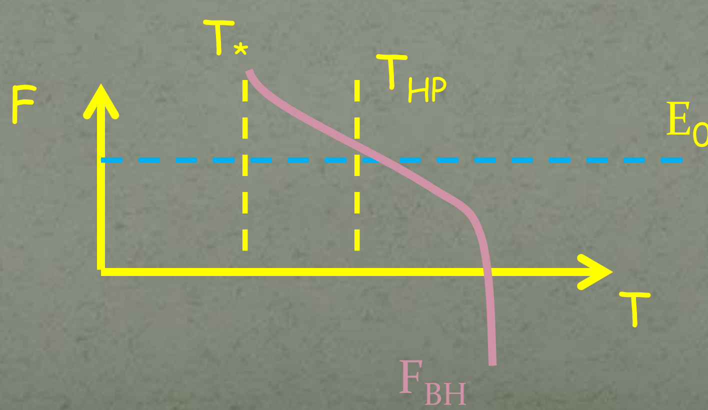


$I = -\ln Z = \beta F = \beta E_{BH} - S_{BH}$

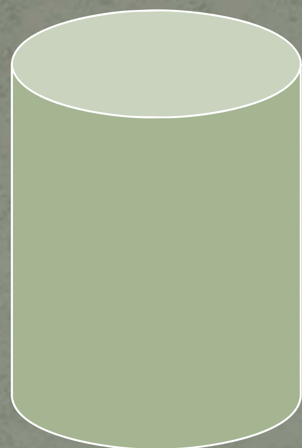
(Hawking & Page)



Since $1/G \sim N^b$, this is a deconfinement transition.
$S = O(1)$ at low T,
$S = O(N^b)$ at large T.

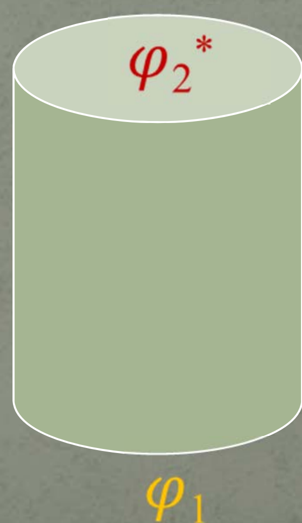
An emergent connection (Maldacena)

Euclidean PI with no identifications



$$= e^{-\beta H}$$

$$= \sum e^{-\beta E} |E><E|$$

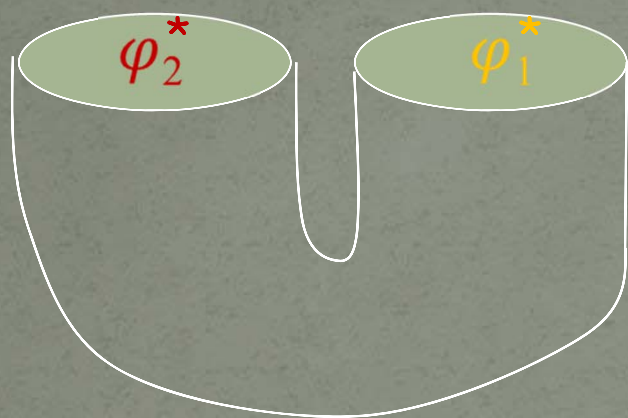


$$= <\varphi_2 | e^{-\beta H} | \varphi_1>$$

$$= \sum e^{-\beta E} <\varphi_2|E><E|\varphi_1>$$

An emergent connection (Maldacena)

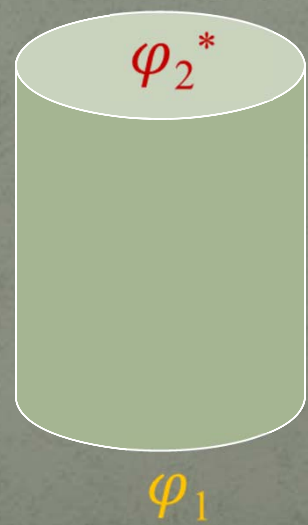
Assume T-reversal symmetry.



$$= \sum e^{-\beta E} < \varphi_2 | E >< E | \varphi_1{}^* >$$

$$= \sum e^{-\beta E} < \varphi_2 | E > \overline{< \varphi_1{}^* | E >}$$

$$= \sum e^{-\beta E} < \varphi_2 | E >< \varphi_1 | E >$$



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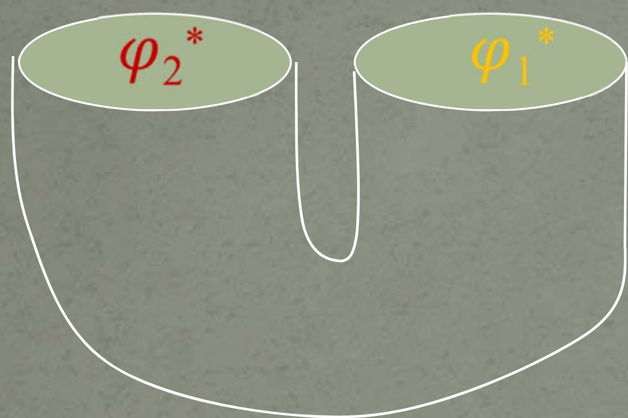

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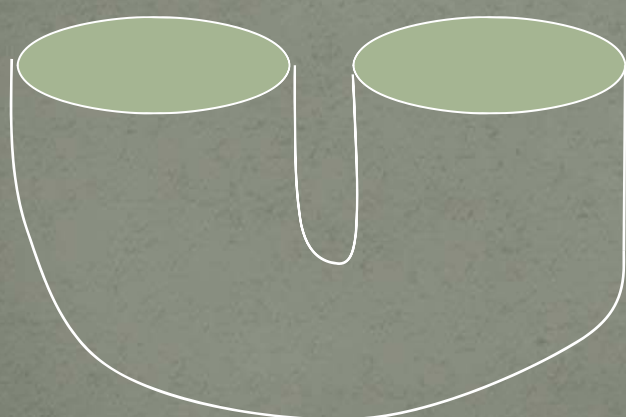


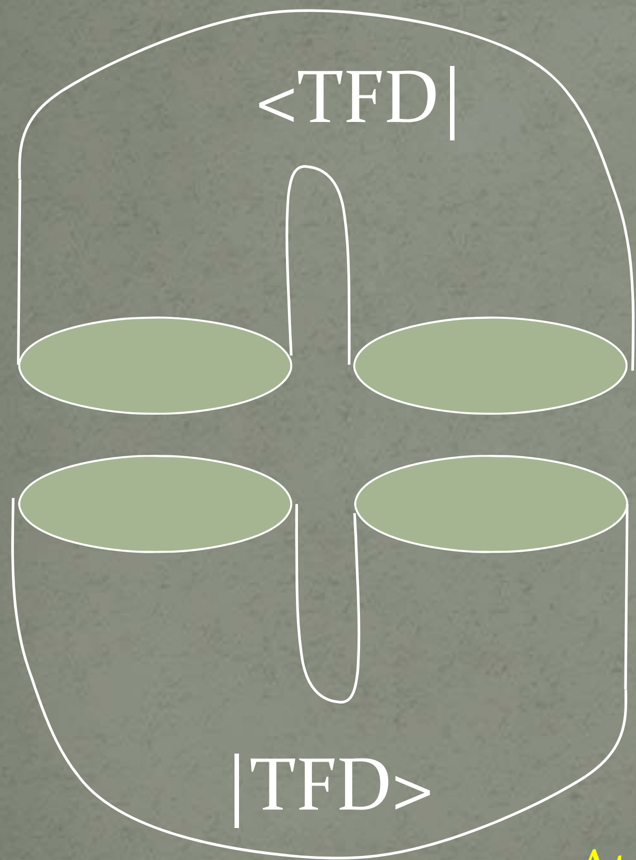
$$= \sum e^{-\beta E} |E > |E > \quad = |TFD >$$

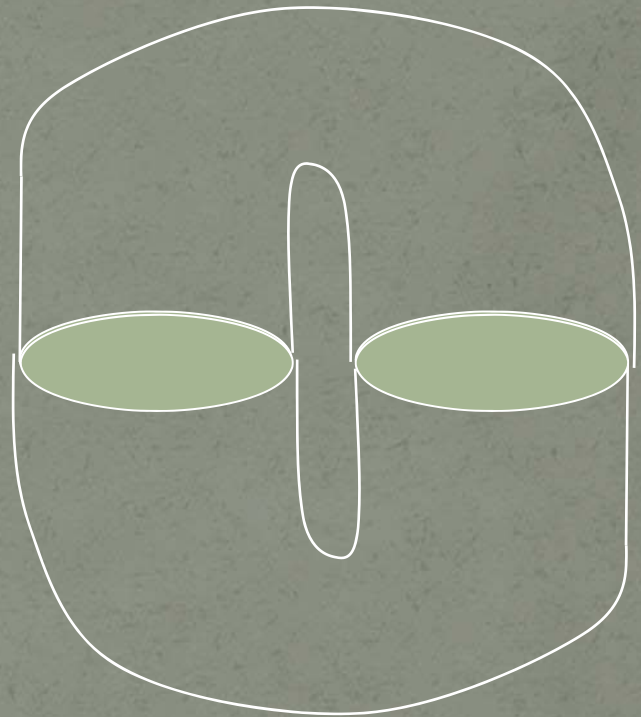
$$\in \mathcal{H}_{CFT} \otimes \mathcal{H}_{CFT}$$

This “thermofield double” state is a joint state on two copies of the CFT!

An emergent connection (Maldacena)

Bulk path integral can be studied by slicing a torus
in half. First consider CFT:





$$= < TFD|TFD >$$

At small β, corresponding bulk path integral is dominated by (large) Euclidean BH saddle point!

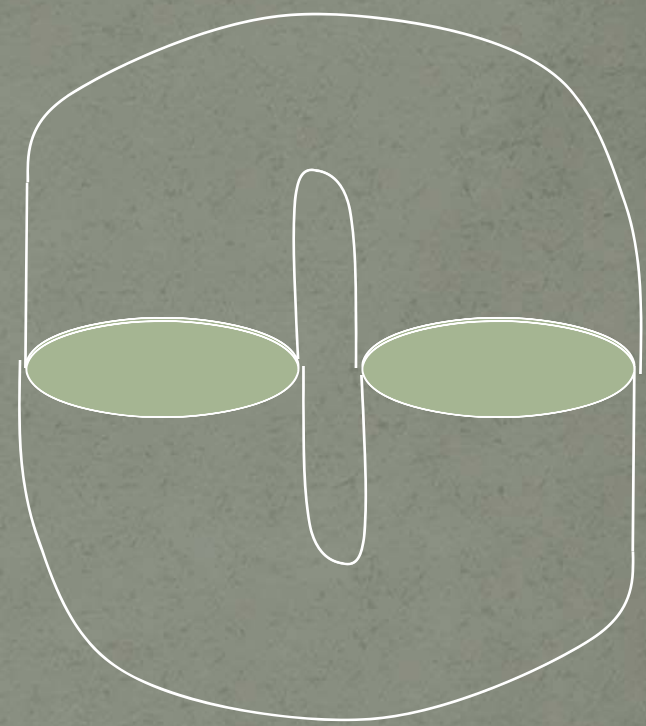
An emergent connection (Maldacena)

Bulk path integral can be studied by slicing a torus in half.

At small β, corresponding bulk path integral is dominated by (large) Euclidean BH saddle point!

Note: t=0 is both a real time and an imaginary (Euclidean) time.

Using the t=0 slice as initial data for a Lorentz-signature spacetime is equivalent to Wick rotation, so |TFD> is dual to a Lorentz-signature black hole with an *exact* time-translation symmetry.

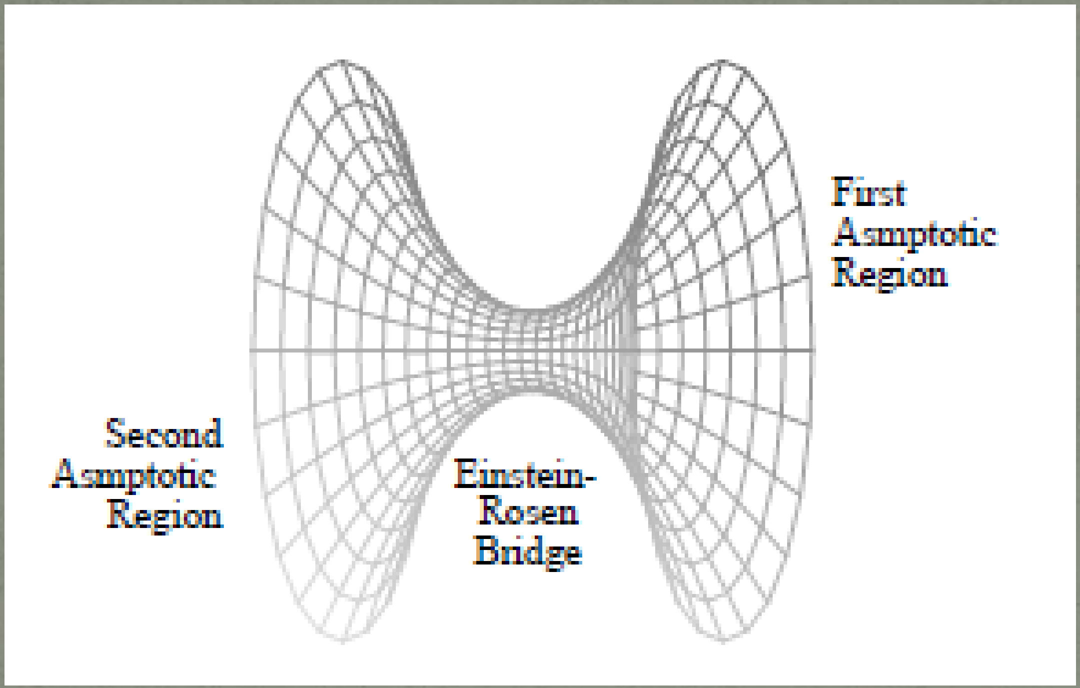


$$= < TFD|TFD >$$

It did not form from collapse at any finite time.
It also has two boundaries.

An emergent connection (Maldacena)

$$|TFD> = \sum e^{-\beta E}|E>|E>$$



Tracing over one side gives a density matrix ρ
with Tr ρln ρ=A/4G.

Entangling the two CFTs in this way gives a geometric connection.

Summary

1. Problems with quantum gravity suggest that both gravity and spacetime emerge from some more fundamental description.

2. AdS/CFT is an example.

3. The thermofield double example suggests that CFT entanglement is related to connectivity in the bulk spacetime.

But wait, there's more…

Open Questions

1. Does strong entanglement always give rise to a geometric connection?
   ER=EPR of Maldacena & Susskind
   Related program by van Raamsdonk, Czech, Karcmarek, Nogueira.

2. How general is the relation between entanglement and area?
   Static conjecture by Ryu & Takayanagi,
   Extened for time-dependence by Hubeny, Rangamani, & Takayanagi,
   "Proof" by Lewkowycz & Maldacena,
   Extended to more general gravity theories by Dong, Camps.

3. Relating entanglement to geometry in perturbed TFDs
   Shenker & Standford, Susskind & Stanford

4. What does this teach us about the duality?
   S=A/4G recently used to derive bulk equations of motion by
   Lashkari, McDermott, & van Raamdonk, extended by
   Faulkner, Guica, Hartman, Myers, & van Raamsdonk,
   Single & van Raamsdonk

   + others!!!

5. Exact duality or leftover bits?
   Challenges by Giddings
   BH interiors suggest more than one bulk state for each CFT state? (Marolf & Wall)

6. Examples more like our universe? (Silverstein, Dong, & Co.)